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Fuzzy Model for Estimating the Worsening of Pathologies Due to Delays in Treatment

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Based on accessible data such as the death registry, this work develops a methodology to provide an estimate of the number of patients and the level of aggravation of their pathologies due to delays in treatment. Firstly, for a given pathology, the deaths will be classified by the most common causes of death. The equivalent number of deceased patients can be obtained by adding this information through the Majority Ordered Weighted Average (MA-OWA). This aggregation will allow obtaining matrix C that indicates the incidence of delay in medical healthcare for each cause of death. Next, matrix L has been obtained, showing the nominal level for each type of patient whose pathology has been aggravated due to a delay in medical attention in each period. From matrices L and C, it is possible to obtain the matrix R that shows the fuzzy relationship between them. The worsening patients in a future period can be obtained from matrix L (obtained from matrix C of that future period and the previously calculated matrix R). Finally, an example illustrates the proposed theoretical model.

Keywords: Fuzzy model, MA-OWA, delay in pathology treatment, shortcomings, healthcare attention, costs

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1 INTRODUCTION

The COVID-19 pandemic has spotlighted a problem that exists in most countries to a greater or lesser extent: the worsening of illnesses due to delays in treatment. The quantification of the incidence of these delays in the

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pathologies of the patients is complex and can be imprecise. For this reason, it is advisable to use techniques that consider the subjectivity and uncertainty existing in the estimates made. Uncertainty decision theory is an area of social sciences and science that has always aroused great interest. Among the pioneers, we can mention authors such as Zadeh [1], who introduced the concept of fuzzy sets, Yager [2], who developed the concept of Ordered Weighted Averaging (OWA) operator, and Kaufmann and Gil-Aluja [3,4]. Furthermore, Peláez and Doña [5, 6] introduced the Majority additive-ordered weighting averaging (MA-OWA), and the linguistic aggregation of majority additive operator (LAMA).

The aim of this work is, based on the registration of deaths caused by a particular disease, to develop a model that makes it possible to estimate the patients whose pathology is affected by the delay in treatment. For this, and after a brief review of the basic concepts, a new methodology is proposed that combines different tools that fuzzy logic offers. To illustrate the developed model, the work ends with an example of its application.

2 PRELIMINARIES CONCEPTS

Definition 1. An OWA operator [2] of dimension n is a mapping $F_{OWA}:\mathbb{R}^n \to \mathbb{R}$ that has an associated weighting vector $W=\left[\omega_1,\omega_2,...,\omega_n\right]$ such that $\omega_i \in \left[0,1\right]$ and $\sum_{i=1}^{n}\omega_i = 1$ defined as:

$$F_{OWA}\left(a_1,a_2,...,a_n\right)=\sum_{i=1}^{n}\omega_i a_{\sigma(i)} \tag{1}$$

where $a_{\sigma(i)}$ is the argument value a_i being ordered in non-increasing order, $a_{\sigma(i)} \ge a_{\sigma(i+1)}$.

The OWA operator is a non-linear function of elements since, it implies an ordering process. It presents the properties of symmetry, monotonicity, boundedness, and idempotency.

In this sense, Yager [2, 7] defines the weights of OWA operators from linguistic quantifiers Q based on non-decreasing monotonous functions:

$$\omega_i = Q\left(\frac{i}{n}\right)-Q\left(\frac{i-1}{n}\right),\ i=1,2,...,n \tag{2}$$

Where ω_i represents the increase in satisfaction when passing from $i-1$ to i.

Definition 2. An IOWA operator of dimension n is mapping $F_{IOWA}:\mathbb{R}^n \to \mathbb{R}$ that has an associated weighting vector $W=\left[\omega_1,\omega_2,...,\omega_n\right]$ such that $\omega_i \in \left[0,1\right]$ and $\sum_{i=1}^{n}\omega_i = 1$ defined as:

$$F_{IOWA}\left(\langle a_1,u_1\rangle,\langle a_2,u_2\rangle,\dots,\langle a_n,u_n\rangle\right)=\sum_{i=1}^{n}\omega_i a_{\sigma(i)} \tag{3}$$

where $a_{\sigma(i)}$ is the argument value of pair $\langle a_i,u_i\rangle$ of order inducing variable u_i reordered such that $u_{\sigma(i)} \geq u_{\sigma(i+1)}$. The IOWA operators are all satisfying de the properties of symmetry, monotonicity, boundedness, and idempotency.

Definition 3. Let a set of linguistic labels $S=\{s_0,s_1,\dots,s_{\max}\}$ be uniformly distributed on a scale, then, the ordering is defined as $(s_a,s_b)\in S$, $s_a<s_b \Leftrightarrow a<b$ with s_0 and $s_{\max}$ are the lowest and the highest elements, respectively. The max is given as $|S|-1$ where $|S|$ denotes the cardinality of S.

Definition 4. The conversion of the linguistic labels to the numbers in unit interval $[0,1]$ can be conducted by using the function $Label^{-1}$ defined as $Label^{-1}=S\rightarrow[0,1]$, $Label^{-1}(s_i)=\dfrac{i}{|S|-1}$ with $i=0,1,\dots,\max$. However, the retranslation from the numerical values intro the linguistic labels can be given as $Label(x)=s_i$ for $\dfrac{i}{|S|}\leq x\leq\dfrac{i+1}{|S|}$, $i=0,1,\dots,\max$ and $Label(1)=S_{\max}$

Definition 5. A function $F:S^n\rightarrow S$ is a LOWA operator of dimension n if it present an associated vector W of dimension n with $\omega_j\in[0,1]$ and $\sum_{i=1}^{n}\omega_i=1$ such as:

$$F\left(S_{\alpha_1},S_{\alpha_2},\dots,S_{\alpha_n}\right)=\omega_1\otimes S_{\beta_1}\oplus\omega_2\otimes S_{\beta_2}\oplus\dots\oplus\omega_n\otimes S_{\beta_n}=S_{\bar{A}} \tag{4}$$

Being $\bar{A}=\sum_{j=1}^{n}\omega_j\beta_j$; S_{β_j} is jth smallest of the S_{α_i}

This operator presents the properties symmetry, monotonicity, boundedness, and idempotency.

Definition 6. A neat OWA operator [8] of dimension n is a mapping $F_{NOWA}:\mathbb{R}^n\rightarrow\mathbb{R}$ defined as:

$$F_{NOWA}\left(a_1,a_2,\dots,a_n\right)=\sum_{i=1}^{n}\omega_i\left(a_{\sigma(1)},a_{\sigma(2)},\dots a_{\sigma(n)}\right)a_{\sigma(i)} \tag{5}$$

where $a_{\sigma(i)}$ is the argument value a_i with any permutation and the vector valued function $\omega:\mathbb{R}^n\rightarrow[0,1]^n$ is normalized such that $\sum_{i=1}^{n}\omega_i\left(a_{\sigma(1)},a_{\sigma(2)},\dots a_{\sigma(n)}\right)=1$ being ordered in non-increasing order, $a_{\sigma(i)}\geq a_{\sigma(i+1)}$.

The neat OWA presents the properties symmetry, boundedness, and idempotency. However, many times monotonicity is lost. One example is the majority OWA (MA-OWA)

Definition 7. Being $S=\left\{s_{\alpha_1},s_{\alpha_2},...,s_{\alpha_n}\right\}$ and $\delta_{\alpha_1},\delta_{\alpha_2},...,\delta_{\alpha_n}\in N$ the label frequency or cardinality, where $\delta_{\alpha_i}>0$, $\delta_{\alpha_i}\leq\delta_{\alpha_{i+1}}$ for every $\alpha_1\leq\alpha_i\leq\alpha_{n-1}$. The linguistic operator MA-OWA [5, 6] is defined as a function $F_{LAMA}:\mathbb{R}^n\times\mathbb{R}^N\rightarrow\mathbb{R}$

$$F_{LAMA}\left(s_1,s_2,...,s_n\right)=\omega_{1,N}\otimes b_{\sigma(1)}\oplus...\oplus\omega_{n,N}\otimes b_{\sigma(n)} \tag{6}$$

where $N=\max_{1\leq i\leq n}\delta_i$ and σ denotes a permutation of group of argument s_i with respect to the cardinality δ_i, such that $b_{\sigma(i)}\geq b_{\sigma(i+1)}$. Moreover, $\otimes$ is the product of a label by a real positive and $\oplus$ is the sum of labels [9]. The weights associated to the arguments are defined by the recurrence relations:

$$\omega_{i,1}=\frac{1}{u_1}=\frac{1}{n}:u_1=n \tag{7}$$

$$\omega_{i,k}=\frac{\gamma_{i,k}+\omega_{i,k-1}}{u_k}:\forall k,2\leq k\leq N \tag{8}$$

where $u_k=1+\sum_{j=1}^{n}\gamma_{j,k}$, and $\sum_{i=1}^{n}\omega_{i,k}=1$ for $k=N$ such that:

$$\gamma_{j,k}=\begin{cases}1 & \delta_{\sigma(j)}\geq k\\ 0 & otherwise\end{cases} \tag{9}$$

3 MATERIAL AND METHODS

3.1 Obtaining Matrix C of Deaths

1. During a period of time $T=\left\{T_t\right\}$, $t=1,...,T$ the leading causes of death are delimited for a given pathology, and information on deaths regarding the worsening diseases is collected. The causes of death will be denoted with C_j, $j=1,...,J$.
2. The relationship between the delay in treatment and the causes of death C_j will be studied, For each cause of death, the effect that the delay has had on death is recorded using linguistic labels $S=\left\{s_0,s_1,...,s_5\right\}$ being $s_0=very$ slight, $s_1=$ slight, $s_2=neutral$, $s_3=$ slightly strong, $s_4=$ strong and $s_5=very_$strong
3. Obtaining the weights to be applied to the linguistic labels per year and cause of death. For this, expressions (7) to (9) apply, where i refers to the linguistic label s_i. Note that because the cardinality vector m changes in each year and causes death, the weights will also.

4. Obtaining the importance of delays for the year t and cause j ($d_{t,j}$) according to expression (6) using the wights obtained in section 3
5. Obtaining the matrix $C = \left[c_{t,j} \right]$ that shows the intensity of the cause of death j in the period t according to the following expression whose denominator reflects the maximum value for the set of years:

$$c_{t,j} = \frac{d_{t,j}}{\max_{t} d_{t,j}} \tag{10}$$

3.2 Obtaining Matrix L of Worsening of Pathologies because of Delays in Treatment

1. During the studied period, $T = \{T_t\}$, $t = 1,...,T$, patients whose pathology has worsened because of a delay in treatment will be estimated (keep in mind that not all patients whose pathology is aggravated die).
2. In order to establish the level of worsening in the disease that produces the delay in medical healthcare, linguistic labels will be used $S = \{s_0, s_1, s_2, s_3\}$ labelling as: $s_0 = \textit{A little}$, $s_1 = \textit{Moderately}$, $s_2 = \textit{Much}$ and $s_3 = \textit{Completely}$ denoting for each year t, the number of patients corresponding to each linguistic label (patient type delimited according to the worsening level) h, $a_{t,h}$.
3. Obtaining annual weights from the expression (2) using the function $Q(t) = t^{\alpha}$:

$$\omega_t = \left(\frac{t}{T}\right)^{\alpha} - \left(\frac{t-1}{T}\right)^{\alpha}, \; i = 1,2,...,T \tag{11}$$

4. Creation of the annual proximity index $p_{t,h}$ of the label s_h as the ratio between the patients corresponding to the label s_h of year t, $a_{t,h}$, and the total patients per year $\sum_{h=0}^{3} a_{t,h}$

$$p_{t,h} = \frac{a_{t,h}}{\sum_{h=0}^{3} a_{t,h}} \tag{12}$$

5. Obtaining the components to be added according to the IOWA defined in, (3), where the weights have been obtained according to (2) being t the year and $\alpha = 0.8$ so that a greater weight is assigned to those years in which the proportion of worsened patients of the considered category over the total is higher. The weights have been $\omega_t = (0.276, 0.205, 0.184, 0.172, 0.163)$

$$F_{IOWA,h}\left(\left\langle a_{1,h},p_{1,h}\right\rangle,\left\langle a_{2,h},p_{2,h}\right\rangle,\ldots,\left\langle a_{T,h},p_{T,h}\right\rangle\right)=\sum_{t=1}^{3}\omega_{t,h}a_{\sigma(t,h)} \quad (13)$$

6. Obtaining matrix $L=\left[l_{t,h}\right]$ as the quotient between element corresponding to year t (13) and the maximum of the IOWA elements for the whole of the T years

$$l_{t,h}=\frac{\omega_{t,h}a_{\sigma(k,h)}}{\max_{t}\omega_{t,h}a_{\sigma(t,h)}} \quad (14)$$

The resulting matrix $L=\left[l_{t,h}\right]$ shows the nominal level of each type of patient whose pathology is worsened by the delay in medical healthcare for each period, being t the period, and h the type of patient determined according to the level of worsening.

3.3 Obtaining Matrix R

The objective is to establish the relationship between matrix of deaths C and the worsening matrix L through the matrix $R=\left[r_{j,h}\right]_{4\times 5}$, with $r_{j,h}\in[0,1]$, $j=1,\ldots,5$ y $h=0,\ldots,3$, starting from the expression:

$$L=C\circ R \quad (15)$$

In expression (15), matrix C of deaths and matrix L of worsening pathologies are fuzzy and known a priori. Matrix $R=\left[r_{j,h}\right]_{5\times 4}$ can be obtained by solving the fuzzy equation:

$$R=C^{-1}\ \alpha\ L=\left[c_{j,t}\right]\ \alpha\ \left[l_{t,h}\right] \quad (16)$$

Being $C=\left[c_{t,j}\right]^{-1}=\left[c_{j,t}\right]$ and $\left[r_{j,h}\right]=\underset{t}{\wedge}\left[c_{j,t}\ \alpha\ \mathrm{l}_{\mathrm{t,h}}\right]$, where

$$c_{j,t}\ \alpha\ \mathrm{l}_{\mathrm{t,h}}=\begin{cases}1 & \textit{if}\ \mathrm{c}_{j,t}\le l_{t,h}\\ c_{j,t} & \textit{if}\ \mathrm{c}_{j,t}>l_{t,h}\end{cases} \quad (17)$$

In this way, the matrix R shows the relationship between deaths and patients whose pathology has been worsened. Therefore, it will predict the number and level of worsening in patient pathologies in future periods through the knowledge of the death produced in them.

3.4 Estimate the Worsening Patients for Levels in a Future Period t+1

While the number of patients who died due to pathology in a future period C^{*} is objective data that only requires access to health records, it is not so easy to obtain data on the number of aggravated patients in that period L^{*}.

Therefore, the estimation of the relationship between both variables will allow us to obtain the number of aggravated patients based on the number of deceased patients.

$$L^* = C^* \circ R \tag{18}$$

Since said matrix provides values between 0 and 1, it is then necessary to estimate the patients with aggravation of each type to multiply them by said matrix and obtain the number of aggravated patients. That's why the matrix number of patients with aggravation in pathologies is estimated $C^* = \left\{c^*_{1,h}\right\}$.

The matrix L^* indicates the degree of belonging of patients to each one of the levels of aggravation in their pathology. If the said degree of membership is multiplied by the maximum number of patients in the group, $\max_t\left(a_{t,h}\right)$, the number of patients in the period $t+1$ for each of the levels of aggravation is obtained. Each of the elements of the matrix C^* is obtained in the following way:

$$c^*_{1(t+1),h} = l^*_{1(t+1),h} \cdot \max_t\left(a_{t,h}\right),\ \mathrm{h} = 0,1,2,3 \tag{19}$$

4 RESULTS

In a particular health area, the causes of death due to the worsening of a specific pathology were recorded for five consecutive years. For each of the causes, a linguistic label $S = \left\{s_0, s_1, ..., s_5\right\}$ was indicated in section 3.1.2. The results are shown in Table 1. As can be seen, for the cause C_1, no one indicated that the cause was very slight or slight, one indicated neutral, two slightly strong, three strong and four very strong.

Since an MA-OWA, a neat OWA, has been used for the aggregation, the weights are different for each year and cause of death, so Table 2 indicates the weights corresponding to each year and cause of death.

As a result of the application of section 3.1, it has been possible to obtain Table 3, which shows the incidence of shortcomings in medical healthcare for

Year	**C$_1$**	**C$_2$**	**C$_3$**	**C$_4$**	**C$_5$**
1	{0; 0; 1; 2; 3; 4}	{0; 0; 1; 1; 2; 3}	{0; 0; 1; 3; 3; 5}	{0; 0; 1; 2; 3; 3}	{0; 0; 0; 1; 1; 2}
2	{0; 0; 2; 2; 3; 5}	{0; 0; 1; 1; 2; 3}	{0; 1; 2; 3; 5; 7}	{0; 1; 1; 3; 3; 4}	{0; 0; 1; 1; 2; 3}
3	{0; 1; 2; 3; 5; 6}	{0; 0; 1; 1; 3; 4}	{0; 0; 2; 2; 5; 8}	{0; 1; 2; 2; 3; 5}	{0; 1; 1; 1; 2; 3}
4	{0; 0; 1; 3; 4; 4}	{0; 0; 1; 1; 2; 2}	{0; 0; 2; 3; 4; 6}	{0; 0; 1; 2; 3; 4}	{0; 0; 0; 1; 2; 2}
5	{0; 0; 1; 1; 2; 3}	{0; 0; 0; 1; 2; 2}	{0; 0; 1; 2; 3; 4}	{0; 0; 1; 1; 2; 2}	{0; 0; 1; 1; 1; 1}

TABLE 1
Impacts of shortcomings in healthcare attention on each cause of death (C_1,...,C_5) and each year

Year	C_1	C_2	C_3	C_4	C_5
1	{0.00; 0.00; 0.01; 0.05; 0.22; 0.72}	{0.00; 0.00; 0.04; 0.04; 0.21; 0.71}	{0.00; 0.00; 0.00; 0.08; 0.08; 0.83}	{0.00; 0.00; 0.02; 0.10; 0.44; 0.44}	{0.00; 0.00; 0.00; 0.17; 0.17; 0.67}
2	{0.00; 0.00; 0.02; 0.02; 0.10; 0.85}	{0.00; 0.00; 0.04; 0.04; 0.21; 0.71}	{0.00; 0.00; 0.00; 0.01; 0.12; 0.87}	{0.00; 0.01; 0.01; 0.16; 0.16; 0.66}	{0.00; 0.00; 0.04; 0.04; 0.21; 0.71}
3	{0.00; 0.00; 0.00; 0.02; 0.24; 0.74}	{0.00; 0.00; 0.01; 0.01; 0.24; 0.74}	{0.00; 0.00; 0.00; 0.00; 0.06; 0.94}	{0.00; 0.00; 0.02; 0.02; 0.10; 0.85}	{0.00; 0.03; 0.03; 0.03; 0.20; 0.70}
4	{0.00; 0.00; 0.01; 0.11; 0.44; 0.44}	{0.00; 0.00; 0.08; 0.08; 0.42; 0.42}	{0.00; 0.00; 0.01; 0.03; 0.11; 0.86}	{0.00; 0.00; 0.01; 0.05; 0.22; 0.72}	{0.00; 0.00; 0.00; 0.11; 0.44; 0.44}
5	{0.00; 0.00; 0.04; 0.04; 0.21; 0.71}	{0.00; 0.00; 0.00; 0.11; 0.44; 0.44}	{0.00; 0.00; 0.01; 0.05; 0.22; 0.72}	{0.00; 0.00; 0.08; 0.08; 0.42; 0.42}	{0.00; 0.00; 0.25; 0.25; 0.25; 0.25}

TABLE 2
Weight matrix

Year	C_1	C_2	C_3	C_4	C_5
1	0.58	0.76	0.65	0.62	0.50
2	0.72	0.76	1.00	0.86	0.89
3	1.00	1.00	0.96	1.00	1.00
4	0.65	0.59	0.83	0.75	0.60
5	0.40	0.51	0.53	0.40	0.39

TABLE 3
Matrix C

each cause of death ($C_1,...,C_5$). First, the equivalent deaths for each year and cause of death were obtained, considering a grouping based on the MA-OWA, and then the proportion that the equivalent deaths of each year represented over the value of the year whose value was maximum. Thus, in matrix C, the value of year 1 for cause A is 0.58, which indicates that, in year 1, the equivalent deceased patients represent 58% of the maximum, for year 2, 72%, and so on.

Table 4 shows matrix L, the importance of each level of yearly worsening caused by the delay in the treatment received. To score said importance, an IOWA was used. The ordering factor was the quotient between the IOWA element corresponding to annually worsened patients for each category and the total sum of the IOWA elements. The weights used have been obtained from (2) using a $\alpha = 0.8$.

The fuzzy relationship between the matrix of deaths C and the matrix of worsening patients has been obtained by applying (16) and (17), and is shown in Table 5.

	Level of worsening			
Year	A little	Moderately	Much	Completely
1	0.72	0.78	0.20	0.31
2	0.51	1.00	0.54	0.51
3	0.33	0.62	1.00	1.00
4	0.59	0.70	0.42	0.61
5	1.00	0.44	0.22	0.26

TABLE 4
Matrix L

	Level of worsening			
	A little	Moderately	Much	Completely
C_1	0.33	0.62	0.20	0.26
C_2	0.33	0.44	0.20	0.26
C_3	0.33	0.44	0.20	0.26
C_4	0.33	0.62	0.20	0.26
C_5	0.33	0.62	0.20	0.26

TABLE 5
Matrix R

	C_1	C_2	C_3	C_4	C_5
Incidence	{0; 0; 1; 1; 2; 2}	{0; 0; 0; 1; 1; 2}	{0; 0; 1; 1; 3; 4}	{0; 0; 0; 1; 1; 2}	{0; 0; 0; 1; 1; 1}
Weights	{0.00; 0.00; 0.08; 0.08; 0.42; 0.42}	{0.00; 0.00; 0.00; 0.11; 0.44; 0.44}	{0.00; 0.00; 0.01; 0.05; 0.22; 0.72}	{0.00; 0.00; 0.08; 0.08; 0.42; 0.42}	{0.00; 0.00; 0.25; 0.25; 0.25; 0.25}

TABLE 6
Incidence of worsening of the initial pathology and weights for year 6

Matrix C^* year 6	{0.31; 0.43; 0.48; 0.29; 0.33}
Matrix L^*	{0.20; 0.41; 0.19; 0.25}
Maximum number of patients per degree	{1,200; 1,087; 947; 450}
Worsening patients by type	{242; 449; 183; 110}

TABLE 7
Worsening patients by type (level of worsening)

Lastly, for the estimation of patients with worsened pathology for year 6 $(T+1)$, the incidence of worsening of the initial pathology was used, and the weights for the application of the MA-OWA were obtained from (7) to (9) (Table 6) which has allowed us to get the matrix C^*. By applying (15) to (17) , matrix L^* has been obtained. Finally, we get the product of said matrix by the maximum number of patients worsened during five years, for each kind of patient (according to the level of aggravation). This product makes it possible to obtain the worsening patients by the delay in treatment. In this case, as shown in Table 7, 242 little worsened patients have been obtained, 449 moderately worsened, 183 much worsened, and 110 completely worsened.

5 CONCLUSIONS

This work makes it possible to estimate the patients and level of worsening in their pathologies in future periods through the knowledge of the deaths (note that not all patients whose pathology is aggravated die). To do this, in relation to a certain pathology, during a study period, the deaths caused by various causes are recorded, assessing the degree to which the delay in medical care has been the trigger for death. This assessment is done with linguistic tags, added using majority OWAs. This process rewards the cases with the greatest number of elements over others with a smaller number of elements (or cardinality). By operating in this way, it is possible to assess the incidence of delays by year and cause of death, which by annual comparison with respect to the maximum for each cause of death makes it possible to obtain matrix C of deaths.

On the other hand, during the same study period, the annual incidence of the number of aggravated patients is obtained (regardless of whether they finally die or not). Types of patients will be delimited according to the level of aggravation experienced in their pathologies based on certain linguistic labels. The annual aggregation is carried out using the weights of an IOWA in which the induced variable is the annual quotient between the number of aggravated patients for said label and the total number of patients. Weights are obtained using an increasing monotonic function. The matrix L of patients aggravated because of delays in treatment reflects the quotient of said values between the maximum for each linguistic label.

Finally, from the L and C matrices it is possible to obtain the matrix R that shows the fuzzy relationship between them. For this reason, it will be possible to predict the number and level of aggravation in the pathologies of patients in future periods through the knowledge of the deaths produced in them.

The main interest of this work is found in the transcendence of the topic it deals with. The aggravation in certain pathologies caused by the delay in their treatment not only supposes an unfortunate human cost, sometimes irreparable, it can also cause a significant economic cost. The proposed model provides a good part of the inputs necessary to assess cost overrun in health care due to delays in treatment. Indeed, once a prediction has been made of the patients whose pathologies have been aggravated, it would be possible to quantify the cost overrun for the healthcare system. For this, an equivalence table will be created to allocate a previously established assessment to each level of worsening pathology. In order to determine the defined standards a priori, the opinion of several experts in the subject will be considered. The experts will be asked to make an assessment of the increase in the average cost per patient arising from the treatment of the analyzed pathology according to the level of worsening condition. Given the subjectivity inherent in the information provided by the experts, we propose the use of the expertise tools introduced by Kaufmann [10] and Kaufmann and Gil Aluja [11]. In this way, an interesting line of research is opened that will allow us to expand our model, also giving it a financial nature.

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